Algebraic Formula Sheet

Arithmetic Operations

\[ ac + bc = c(a + b) \quad \text{and} \quad a\left( b\over c \right) = ab \over c \]

\[ \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \]

\[ \frac{a + c}{b + d} = ad + bc \over bd \]

\[ \frac{a - b}{c - d} = b - a \over d - c \]

\[ \frac{ab + ac}{a} = b + c, \quad a \neq 0 \]

\[ x^n \cdot x^m = x^{n+m} \quad \text{and} \quad x^0 = 1, \quad x \neq 0 \]

\[ (x^n)^m = x^{nm} \quad \text{and} \quad \left( \frac{x^m}{y^n} \right) = \frac{x^m}{y^n} \]

\[ (xy)^n = x^n y^n \quad \text{and} \quad \left( \frac{x}{y} \right)^n = y^n \over x^n \]

\[ x^\frac{m}{n} = \left( x^\frac{1}{n} \right)^m = \left( x^m \right)^{1\over n} \quad \text{and} \quad x^\frac{n}{m} = x^{m-n} \]

\[ \left( \frac{x}{y} \right)^{-n} = \left( \frac{y}{x} \right)^n = y^n \over x^n \quad \text{and} \quad x^{-n} = \frac{1}{x^n} \]

Properties of Radicals

\[ \sqrt[n]{x} = x^{\frac{1}{n}} \quad \text{and} \quad \sqrt[n]{\frac{x}{y}} = \sqrt[n]{x} \over \sqrt[n]{y} \]

\[ \sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y} \quad \sqrt[n]{x^n} = x, \quad \text{if} \ n \ \text{is odd} \]

\[ \sqrt[m]{\sqrt[n]{x}} = \sqrt[n\cdot m]{x} \quad \sqrt[n]{x^n} = \left| x \right|, \quad \text{if} \ n \ \text{is even} \]

Properties of Exponents

\[ |x| = \begin{cases} x & \text{if} \ x \geq 0 \\ -x & \text{if} \ x < 0 \end{cases} \]

\[ |x| \geq 0 \quad \text{and} \quad |-x| = |x| \]

\[ |xy| = |x||y| \quad \text{and} \quad \left| \frac{x}{y} \right| = \left| \frac{x}{y} \right| \]

\[ |x + y| \leq |x| + |y| \quad \text{Triangle Inequality} \]

\[ |x - y| \geq \left| |x| - |y| \right| \quad \text{Reverse Triangle Inequality} \]

Properties of Inequalities

\[ a < b \quad \text{then} \quad a + c < b + c \quad \text{and} \quad a - c < b - c \]

\[ a < b \quad \text{and} \quad c > 0 \quad \text{then} \quad ac < bc \quad \text{and} \quad \frac{a}{c} < \frac{b}{c} \]

\[ a < b \quad \text{and} \quad c < 0 \quad \text{then} \quad ac > bc \quad \text{and} \quad \frac{a}{c} > \frac{b}{c} \]

Properties of Absolute Value

Distance Formula

Given two points, \( P_A = (x_1, y_1) \) and \( P_B = (x_2, y_2) \), the distance between the two can be found by:

\[ d(P_A, P_B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Number Classifications

Natural Numbers: \( \mathbb{N} = \{1, 2, 3, 4, 5, \ldots \} \)

Whole Numbers: \( \{0, 1, 2, 3, 4, 5, \ldots \} \)

Integers: \( \mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \} \)

Rationals: \( \mathbb{Q} = \{ \text{All numbers that can be written as a fraction with an integer numerator and a nonzero integer denominator, } \frac{a}{b} \} \)

Irrationals: \( \{ \text{All numbers that cannot be expressed as the ratio of two integers, for example } \sqrt{5}, \sqrt{27}, \text{and } \pi \} \)

Real Numbers: \( \mathbb{R} = \{ \text{All numbers that are either a rational or an irrational number} \} \)


Logarithms and Log Properties

Definition

\[ y = \log_b x \quad \text{is equivalent to} \quad x = b^y \]

Example

\[ \log_2 16 = 4 \quad \text{because} \quad 2^4 = 16 \]

Special Logarithms

\[ \ln x = \log_e x \quad \text{natural log} \]

where \( e = 2.718281828... \)

\[ \log x = \log_{10} x \quad \text{common log} \]

Logarithm Properties

\[ \log_b b = 1 \]
\[ \log_b b^x = x \]
\[ y^{\log_b x} = x \]
\[ \ln e^x = x \]
\[ e^{\ln x} = x \]

\[ \log_b (x^k) = k \log_b x \]
\[ \log_b (xy) = \log_b x + \log_b y \]
\[ \log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y \]

Factoring

\[ xa + xb = x(a + b) \]
\[ x^2 - y^2 = (x + y)(x - y) \]
\[ x^2 + 2xy + y^2 = (x + y)^2 \]
\[ x^2 - 2xy + y^2 = (x - y)^2 \]
\[ x^3 + 3x^2y + 3xy^2 + y^3 = (x + y)^3 \]
\[ x^3 - 3x^2y + 3xy^2 - y^3 = (x - y)^3 \]
\[ x^3 + y^3 = (x + y) \left( x^2 - xy + y^2 \right) \]
\[ x^3 - y^3 = (x - y) \left( x^2 + xy + y^2 \right) \]
\[ x^{2n} - y^{2n} = (x^n - y^n) \left( x^n + y^n \right) \]

If \( n \) is odd then,
\[ x^n - y^n = (x - y) \left( x^{n-1} + x^{n-2}y + ... + y^{n-1} \right) \]
\[ x^n + y^n = (x + y) \left( x^{n-1} - x^{n-2}y + x^{n-3}y^2 ... - y^{n-1} \right) \]

Linear Functions and Formulas

Examples of Linear Functions

\[ y = x \]
\[ y = 1 \]
Constant Function

This graph is a horizontal line passing through the points \((x, c)\) with slope \(m = 0\):

\[ y = c \quad \text{or} \quad f(x) = c \]

Slope \((\text{a.k.a Rate of Change})\)

The slope \(m\) of the line passing through the points \((x_1, y_1)\) and \((x_2, y_2)\) is:

\[
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}
\]

Linear Function/Slope-intercept form

This graph is a line with slope \(m\) and \(y\) - intercept \((0, b)\):

\[ y = mx + b \quad \text{or} \quad f(x) = mx + b \]

Point-Slope form

The equation of the line passing through the point \((x_1, y_1)\) with slope \(m\) is:

\[ y = m(x - x_1) + y_1 \]

Quadratic Functions and Formulas

Examples of Quadratic Functions

- \(y = x^2\) \(\text{parabola opening up}\)
- \(y = -x^2\) \(\text{parabola opening down}\)

Forms of Quadratic Functions

\[
\text{Standard Form} \quad y = ax^2 + bx + c \\
\text{or} \quad f(x) = ax^2 + bx + c
\]

This graph is a parabola that opens up if \(a > 0\) or down if \(a < 0\) and has a vertex at

\[
\left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right).
\]

\[
\text{Vertex Form} \quad y = a(x - h)^2 + k \\
\text{or} \quad f(x) = a(x - h)^2 + k
\]

This graph is a parabola that opens up if \(a > 0\) or down if \(a < 0\) and has a vertex at \((h, k)\).
Quadratics and Solving for $x$

**Quadratic Formula**

To solve $ax^2 + bx + c = 0, \ a \neq 0$, use:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$ 

**The Discriminant**

The discriminant is the part of the quadratic equation under the radical, $b^2 - 4ac$. We use the discriminant to determine the number of real solutions of $ax^2 + bx + c = 0$ as such:

1. If $b^2 - 4ac > 0$, there are two real solutions.
2. If $b^2 - 4ac = 0$, there is one real solution.
3. If $b^2 - 4ac < 0$, there are no real solutions.

**Square Root Property**

Let $k$ be a nonnegative number. Then the solutions to the equation $x^2 = k$ are given by $x = \pm \sqrt{k}$.

**Other Useful Formulas**

**Compound Interest**

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

where:
- $P =$ principal of P dollars
- $r =$ interest rate (expressed in decimal form)
- $n =$ number of times compounded per year
- $t =$ time

**Continuously Compounded Interest**

$$A = Pe^{rt}$$

where:
- $P =$ principal of P dollars
- $r =$ interest rate (expressed in decimal form)
- $t =$ time

**Circle**

$$(x - h)^2 + (y - k)^2 = r^2$$

This graph is a circle with radius $r$ and center $(h, k)$.

**Ellipse**

$$(x - h)^2 + \frac{(y - k)^2}{b^2} = 1$$

This graph is an ellipse with center $(h, k)$ with vertices $a$ units right/left from the center and vertices $b$ units up/down from the center.

**Hyperbola**

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

This graph is a hyperbola that opens left and right, has center $(h, k)$, vertices $(h \pm a, k)$; foci $(h \pm c, k)$, where $c$ comes from $c^2 = a^2 + b^2$ and asymptotes that pass through the center $y = \pm \frac{b}{a}(x - h) + k$.

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

This graph is a hyperbola that opens up and down, has center $(h, k)$, vertices $(h, k \pm a)$; foci $(h, k \pm c)$, where $c$ comes from $c^2 = a^2 + b^2$ and asymptotes that pass through the center $y = \pm \frac{a}{b}(x - h) + k$.

**Pythagorean Theorem**

A triangle with legs $a$ and $b$ and hypotenuse $c$ is a right triangle if and only if

$$a^2 + b^2 = c^2$$